GENERALISATION OF CANONICAL NUMBER SYSTEMS

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Let $\mathcal{E}$ a commutative Ring with identity and $\mathcal{R} = \mathcal{E}/(P)$ with

$$P(x) = p_d x^d + \cdots + p_1 x + p_0 \in \mathcal{E}[x] \qquad (d \in \mathbb{N}_0).$$

Denote by X the image of x under the canonical epimorphism and $\mathcal{N}$ a set of representatives of $\mathcal{R}/(X)$. An $A \in \mathcal{R}$ has a finite X-ary representation if for each $A \in \mathcal{R}$ there exists $e_0, \dots e_h \in \mathcal{N}$ with

$$A = \sum_{i=0}^{h} e_i X^i.$$

We call $(\mathcal{R}, X, \mathcal{N})$ a digit systems if each $A \in \mathcal{R}$ has a finite X-ary representation. The case when $\mathcal{E} = \mathbb{Z}$, P monic and $\mathcal{N} = \{0, \dots, |p_0| - 1\}$ corresponds to the well known Canonical Number Systems (CNS).

First we present some general results concerning this topic. The problem for non monic P is that $\mathcal{R}$ is no finitely generated $\mathcal{E}$ module. For $\mathcal{N} \subset \mathcal{E}$ we will explicitly define a finitely generated d-dimensional $\mathcal{E}$ module $K(\mathcal{R}) \subset \mathcal{R}$ such that $(\mathcal{R}, X, \mathcal{N})$ is a digit system if and only if each $R \in K(\mathcal{R})$ admits a finite X-ary representation. We will use this result in order to generalise known enumeration systems such as CNS and state criteria for them whether to give rise to a digit system or not.

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