FRACTAL TILES ASSOCIATED TO GENERALISED RADIX REPRESENTATIONS AND SHIFT RADIX SYSTEMS

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For $\mathbf{r} \in \mathbb{R}^d$ define the mapping

$$\tau_{\mathbf{r}} : \mathbb{Z}^d \to \mathbb{Z}^d, \mathbf{x} \mapsto (x_1, \dots x_{d-1}, -\lfloor \mathbf{r} \cdot \mathbf{x} \rfloor) \quad (\mathbf{x} = (x_0, \dots, x_{d-1})).$$

$\tau_{\mathbf{r}}$ is called a shift radix system (SRS for short) if for every $\mathbf{z} \in \mathbb{Z}^d$ there exists a $k \in \mathbb{N}$ such that $\tau_{\mathbf{r}}^k(\mathbf{z}) = \mathbf{0}$. It is well known that SRS can be used to describe beta expansions and canonical number systems. In our research we associate to every $\mathbf{r} \in \mathbb{R}^{\mathbf{d}}$ a family of fractal tiles $(\mathcal{T}_{\mathbf{r}}(\mathbf{z}))_{\mathbf{z} \in \mathbb{Z}^d}$ (SRS tiles) with

$$\mathcal{T}_{\mathbf{r}}(\mathbf{z}) := \lim_{n \to \infty} R_{\mathbf{r}}^n \tau_{\mathbf{r}}^{-n}(\mathbf{z})$$

where $R_{\mathbf{r}}$ is the $n \times n$ companion matrix with $-\mathbf{r}$ as last row vector. We present several properties of these SRS tiles and show the relation to tiles induced by Pisot numbers and self affine tiles associated to expanding polynomials.

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