FRACTAL TILES ASSOCIATED TO GENERALISED RADIX REPRESENTATIONS AND SHIFT RADIX SYSTEMS

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For $\mathbf{r} \in \mathbb{R}^{\mathbf{d}}$ define the mapping

$$\tau_{\mathbf{r}} : \mathbb{Z}^d \to \mathbb{Z}^d, \mathbf{x} = (x_1, \dots, x_d) \mapsto (x_2, \dots, x_d, -\lfloor \mathbf{r} \cdot \mathbf{x} \rfloor).$$

$\tau_{\mathbf{r}}$ is called a shift radix system (SRS) if $\forall \mathbf{x} \in \mathbb{Z}^d \, \exists n \in \mathbb{N} : \tau_{\mathbf{r}}^n(\mathbf{x}) = \mathbf{0}$. Shift radix systems are a class of dynamical systems, introduced in 2005 by Akiyama *et al.*, and are strongly related to other well known notions of number systems as β-expansion or canonical number systems. Let

$$\mathcal{D}_d := \left\{ \mathbf{r} \in \mathbb{R}^{\mathbf{d}} \,|\tau_{\mathbf{r}} \text{ is ultimately periodic} \right\} \text{ and}$$
$$\mathcal{D}_d^0 := \left\{ \mathbf{r} \in \mathbb{R}^{\mathbf{d}} \,|\tau_{\mathbf{r}} \text{ is an SRS} \right\}.$$

Further denote by $R(\mathbf{r})$ the companion matrix with the characteristic polynomial $x^d + r_{d-1}x^{d-1} + \cdots + r_0$. For $\mathbf{r} \in \operatorname{int} \mathcal{D}_d$, $\mathbf{x} \in \mathbb{Z}^d$ define

$$T_{\mathbf{r},n}(\mathbf{x}) = \left\{ \mathbf{x} \in \mathbb{Z}^d \,|\, \tau_{\mathbf{r}}^n \mathbf{z} = \mathbf{x} \right\}$$

and

$$T_{\mathbf{r}}(\mathbf{x}) = \lim_{n \to \infty} R(\mathbf{r})^n T_{\mathbf{r},n}(\mathbf{x})$$

(using the limit with respect to a Hausdorff metric). Then $T_{\mathbf{r}}(\mathbf{x})$ is called an *SRS-tile*. We will give basic properties of such tiles and see, how they are related to tiles induced by Pisot numbers and canonical number systems.

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