THREE DIMENSIONAL SYMMETRIC SHIFT RADIX SYSTEMS

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For $\mathbf{r} \in \mathbb{R}^{\mathbf{d}}$ define the mapping

$$\tau_{\mathbf{r},\varepsilon} : \mathbb{Z}^d \to \mathbb{Z}^d, \mathbf{x} = (x_1, \ldots, x_d) \mapsto (x_2, \ldots, x_d, -\lfloor \mathbf{r} \cdot \mathbf{x} + \varepsilon \rfloor).$$

$\tau_{\mathbf{r}}$ is called an ε-shift radix system (ε-SRS) if for all $\mathbf{x} \in \mathbb{Z}^d$ there exists an $n \in \mathbb{N}$ with $\tau_{\mathbf{r},\epsilon}^n(\mathbf{x}) = \mathbf{0}$. Originally shift radix systems have been introduced by Akiyama *et al.* with $\varepsilon = 0$. They are strongly related to other well known notions of number systems as β-expansion or canonical number. We will concentrate on the symmetric case ($\varepsilon = \frac{1}{2}$), which was the first time treated by Akiyama and Scheicher. Let

$$\mathcal{D}_d(\varepsilon) := \left\{ \mathbf{r} \in \mathbb{R}^d \,|\, \tau_{\mathbf{r},\varepsilon} \text{ is ultimately periodic} \right\} \text{ and}$$
$$\mathcal{D}_d^0(\varepsilon) := \left\{ \mathbf{r} \in \mathbb{R}^d \,|\, \tau_{\mathbf{r},\varepsilon} \text{ is an } \epsilon\text{-SRS} \right\}.$$

The sets $\mathcal{D}_d(\varepsilon)$ are, except for the boundaries, easy to describe for all $\varepsilon \in [0, \frac{1}{2}]$ while the sets $\mathcal{D}_d^0(\epsilon)$ have a quite complicated structure, at least for $\varepsilon = 0$. Here, apart from the trivial cases $d = 0, 1$, exists only approximations for $d = 2$. In the symmetric case, the situation becomes more clearly. $\mathcal{D}_2^0(\frac{1}{2})$ has been fully analysed by Akiyama and Scheicher. We will supplement this result by giving a full characterisation of $\mathcal{D}_3^0(\frac{1}{2})$.

Supported by FWF Project Nr. P9610-N13.